17P405 (Pages: 2) Name……………………

Reg.No…………………..

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCSS - PG)

(Mathematics)

CC17P MT4 E01 - COMMUTATIVE ALGEBRA

(2017 Admission Regular)

Time : Three Hours Maximum:36 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define a semi-local ring and give an example.
2. Define the nilradical and Jacobson radical of a ring A.
3. Define co-prime ideal and give an example.
4. Show that when the radical of two ideals are co-prime, then the ideals are co-prime.
5. State Nakayamma's lemma.
6. Show that the A-module $S^{-1}A$ is a flat A-module.
7. Define **p**-primary ideal in a ring and give an example.
8. State first uniqueness theorem.
9. Define local property of an A-module and give an example.
10. If η is the nilradical of A, then show that the nilradical of $S^{-1}A$ is $S^{-1}\eta$
11. State "Going-up theorem".
12. Define a valuation ring and give an example.
13. Define Noetherian and Artinian rings.
14. State Structure theorem for Artin rings.

(14 x 1 = 14 Weightage)

PART B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Show that every ring A $\neq$ 0 has at least one maximal ideal.
16. Show that the nilradical of a ring A is the intersection of all prime ideals of A
17. Show that M is a finitely generated A-module if and only if M is isomorphic to a quotient of A^n for some integer n > 0
18. Let $M' \xrightarrow{f} M \xrightarrow{g} M'' \rightarrow 0$ be an exact sequence of A-modules and homomorphisms, and N be any A-module, then show that the sequence $M' \otimes N \xrightarrow{f \otimes 1} M \otimes N \xrightarrow{g \otimes 1} M'' \otimes N \rightarrow 0$ is exact.

19. Let $M^{'} \xrightarrow{f} M \xrightarrow{g} M^{"}$ is exact at M. Then show that

$S^{-1}M^{'} \xrightarrow{s^{-1}f} S^{-1}M \xrightarrow{s^{-1}g} S^{-1}M^{"}$ is exact at $S^{-1}M$

20. Show that the primary ideals in Z are (0) and (p^n) , where p is prime.
21. State and prove second uniqueness theorem.
22. State and prove "Going-down theorem" .
23. Show that M is a Noetherian A-module if and only if every submodule of M is finitely generated.
24. If A is Noetherian, then show that the polynomial ring A[x] is Noetherian.

(7 x 2 = 14 Weightage)

PART C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Let $M^{1} \xrightarrow{u} M \xrightarrow{v} M^{11} \rightarrow 0$ be an exact sequence of A-modules an homomorphisms. Then show that the sequence is exact if and only if for all A-modules N, the sequence

$0 \rightarrow Hom(M^{11}, N) \xrightarrow{\bar{v}} Hom(M, N) \xrightarrow{\bar{u}} Hom(M^{1}, N)$ is exact.

26. Explain the construction of rings of fractions.
27. Let a be a decomposable ideal, let $a = \bigcap_{i=1}^{n} q_i$ be a minimal primary decomposition, and let $r(q_i) = p_i$. Then show that $\bigcup_{i=1}^{n} p_i = \{x \in A : (a : x) \neq a\}$
28. Let (B, g) be a maximal element of Σ. Then show that B is a valuation ring of the field k .

(2 x 4 = 8 Weightage)

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