17P406 (Pages: 2) Name…………………….

Reg. No………………..

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCSS - PG)

(Mathematics)

CC17P MT4 E07 – ADVANCED FUNCTIONAL ANALYSIS

(2017 Admission Regular)

Time: Three Hours Maximum: 36 Weightage

PART - A

Answer ***all*** questions. Each question carries 1 weightage.

1. Show that if X is a finite dimensional normed space then its dual has the same dimension.
2. Show that the dual of c_{00} with norm $\|.\|_p$ is linearly isometric to l^q
3. Let X and Y be normed spaces and $F \in BL(X, Y)$. Then prove that
$$Z(F) = \{x \in X: x'(x) = 0, \forall\ x' \in R(F')\}$$
4. Define Moment sequences. Show that for $Z \in BV([0,1])$, $|\mu(n)| \leq V(Z)$
5. Let X be a finite dimensional normed space. Show that $x_n \xrightarrow{w} x$ if and only if $x_n \to x$
6. Using an example show that $x'_n \xrightarrow{w^*} x'$ need not imply $x'_n \to x'$ in X
7. Every bounded sequence in X' need not have a weak * convergent subsequence. Give an example.
8. Define a reflexive space. Show that every closed subspace of X is reflexive.
9. Show that for $1 \leq p < \infty, l^p$ is reflexive.
10. A strictly convex normed space is uniformly convex. True or False. Justify your answer.
11. Show that for $A \in CL(X)$, where X is a normed space,$\sigma(A') = \sigma(A)$
12. Let X be an inner product space and $u_i \in X'$ for i=1, 2, …… If $\{u_1, u_{2,} \dots.\}$ is an orthonormal set in X, show that $\sum_n |f(u_n)|^2 \leq \|f\|^2$
13. Let (x_n) be a sequence in Hilbert space H. Prove that $x_n \to x$ if and only if $x_n \xrightarrow{w} x$ and $\limsup_{n\to\infty} \|x_n\| \leq \|x\|$
14. Let H be a Hilbert space and $A \in BL(H)$. Show that there exists a unique $B \in BL(H)$ such that $\forall x, y \in H$; $< A(x), y > = < x, B(y) >$

(14 x 1 = 14 Weightage)

PART- B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Prove that if X' is seperable then so is X
16. Prove: If X be a normed space and $A \in BL(X)$, then $\sigma(A') \subset \sigma(A)$ and if X is a Banach space, then $\sigma(A) = \sigma_a(A) \cup \sigma_e(A') = \sigma(A')$
17. Let $Z \in BV([a,b])$, show that there exists unique $y \in NBV([a,b])$ such that $\int_a^b xdz = \int_a^b xdy$ for every $x \in C([a,b])$ and $V(y) \leq V(z)$
18. Let $\mu(n)$ be a sequence of scalars. Show that $\mu(n)$ is a moment sequence, then $\sum_{j=0}^{n} |\alpha_{n,j}| \leq \alpha, \forall n = 0,1,2 \ldots$ and $\alpha > 0$ where $\alpha_{n,j} = \binom{n}{j} (-1)^{n-j} \Delta^{n-j} \mu(j)$
19. State and prove Schur's lemma.
20. Let X be a normed space and $\{x_1', x_2', \ldots . x_m'\}$ be a linearly independent subset of X' Then prove that there are $x_1, x_2 \ldots \ldots x_m$ in X such that $x_j'(x_i) = \delta_{ij}$
21. Let X be a uniformly convex normed space and (x_n) be a sequence in X such that $\|x_n\| \to 1$ and $\|x_n + x_m\| \to 2$ as $n, m \to \infty$. Then show that $\lim_{n,m\to\infty} \|x_n - x_m\| = 0$
22. Let X be a normed space and $A \in CL(X)$ Then show that $dim[Z(A' - kI)] = \dim[Z(A - kI)] < \infty$ for $0 \neq k \in K$
23. State and prove Unique Hahn Banach Extension theorem.
24. Let H be a Hilbert space and $A \in BL(H)$. Show that $R(A) = H$ if and only if A^* is bounded below and $R(A^*) = H$ if and only if A is bounded below.

(7 x 2 = 14 Weightage)

PART-C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Let $1 \leq p \leq \infty$ and $\frac{1}{p} + \frac{1}{q} = 1.$ Define $f_y: L^p \to K$ by $f_y(x) = \int_a^b xydm$ then show that $f_y \in (L^p)'$ and $\|f_y\| = \|y\|_q$. Also prove that the map $F: L^q \to (L^p)'$ defined by $F(y) = f_y; y \in L^q$ is a linear isometry.
26. Let X be a normed space. Then X is reflexive if and only if X is a Banach space and every bounded sequence in X has a weak convergent subsequence.
27. State and prove Riesz representation theorem.
28. Show that a subset of a Hilbert space H is weak bounded if and only if it is bounded.

(2 x 4 = 8 Weightage)

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