17P457 (Pages: 2) Name……….…………….

Reg. No…………………..

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2019

(CUCSS - PG)

(Statistics)

CC15P ST4 C13 – MULTIVARIATE ANALYSIS

(Regular/Improvement/Supplementary)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Distinguish between partial and multiple correlation coefficients.
2. Let $\boldsymbol{Y} \sim N_p(\boldsymbol{\mu}, \boldsymbol{I})$, where $\boldsymbol{Y}$ and $\boldsymbol{\mu}$ are $p \times 1$ vectors and $\boldsymbol{I}$ is a $p \times p$ identity matrix. Show that $(\boldsymbol{Y} - \boldsymbol{\mu})^T(\boldsymbol{Y} - \boldsymbol{\mu}) \sim \chi^2_{(p)}$
3. If $\boldsymbol{X}$ is a random vector with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ and if $\boldsymbol{A}$ is a symmetric matrix of constants, then show that $E(\boldsymbol{X}^T\boldsymbol{A}\boldsymbol{X}) = tr(\boldsymbol{A\Sigma}) + \boldsymbol{\mu}^T\boldsymbol{A\mu}$
4. Write the necessary and sufficient condition for one subset of a multivariate normal random variable and the subset consisting of remaining variables to be independent.
5. Obtain the distribution of sample mean of n observations from $N_p(\boldsymbol{0}, \Sigma)$
6. Explain generalized variance.
7. Consider a multivariate Normal distribution $N_p(\boldsymbol{\mu}, \Sigma)$. Verify whether the MLE estimator of Σ is unbiased or not?
8. Prove that the maximum likelihood estimators of the mean vector and the covariance matrix of a multivariate normal distribution are independent.
9. Why the Wishart distribution does considered as the multivariate analogue of the chi square distribution?
10. Prove that the generalized variance of a random vector is same as that of the vector of its principal components.
11. Define Fisher’s discriminant function.
12. Explain factor rotation.

(12 × 1 = 12 Weightage)

Part B

Answer any ***eight*** questions. Each question carries 2 weightage.

13. If $\boldsymbol{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ derive the distribution of $\boldsymbol{Y} = \mathbf{C}\boldsymbol{X}$, where $\boldsymbol{C}$ is a vector of constants.
14. If $\boldsymbol{X} \sim N_3\left(\begin{pmatrix}2\\3\\4\end{pmatrix}, \begin{bmatrix}4 & 6 & 2\\2 & 5 & 4\\3 & 1 & 3\end{bmatrix}\right)$. Find the conditional distribution of $(X_1, X_3)/ X_2 = 3$
15. Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be symmetric matrices of constants. If $\boldsymbol{Y} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ derive the necessary and sufficient condition for the independence of $\boldsymbol{Y}^T\boldsymbol{AY}$ and $\boldsymbol{Y}^T\boldsymbol{BY}$
16. Derive the maximum likelihood estimator of the covariance matrix of a multivariate normal distribution.
17. Show that Hotelling's T^2 statistic is invariant under linear transformation.
18. Derive the sampling distribution of multiple correlation coefficient of a multivariate normal distribution when the population multiple correlation coefficient is zero.
19. Explain sphericity test.
20. If the m-component vector $\boldsymbol{Y}$ is distributed according to $N(\boldsymbol{v}, \boldsymbol{T})$, then), then prove that $\boldsymbol{Y}^T\boldsymbol{TY}$ is distributed according to the non-central chi square distribution with m degrees of freedom.
21. State and prove the additive property of Wishart distribution.
22. How would you estimate the canonical correlation and canonical variables?
23. Explain the classification problem with a suitable example.
24. Explain principal component analysis.

(8 × 2 = 16 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. Derive the probability density function of a p dimensional multivariate normal distribution.
26. Obtain the sampling distribution of sample correlation coefficient when the population correlation coefficient is zero.
27. Derive the likelihood ratio test for testing the equality of dispersion matrices of multivariate Normal distributions.
28. Explain the factor analysis. Describe the principal component method for estimating the factor model.

(2 × 4 = 8 Weightage)

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