D 33332 **(Pages : 2)** **Name..................................**

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, FEBRUARY 2013

(CUCSS)

Mathematics

MT 1C 02—LINEAR ALGEBRA

(2010 admissions)

Time : Three Hours **Maximum : 36 Weightage**

Part A (Short Answer Type)

Answer **all** *questions.*
Each question carries weightage 1.

1. Let V be a vector space over a field F. Prove that if $0 \in V$ and $C \in F$ then $C.O = 0$.
2. Prove that $U = \{(x, x) : x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.
3. Verify whether the set of all upper triangular matrices span the space of all 2×2 matrices.
4. Find the dimension of the space of all $n \times n$ diagonal matrices over $\mathbb{R}$.
5. Find the co-ordinate vector of $(1,2,3) \in \mathbb{R}^3$ with respect to the ordered basis $\{(1,2,0), (1,1,0), (0,1,1)\}$.
6. Verify whether $f(x, y) = xy$ for $(x, y) \in \mathbb{R}^2$ is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}$.
7. Let $W = \text{span}\{(1,1,0), (1,0,1)\}$. Let f be defined by $f(x, y, z) = x - y - z$. Verify whether f belongs to W^0.
8. Find the characteristic polynomial of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
9. Find the characteristic values of $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$.
10. Let $W = \text{span}\{(1,1,1)\}$ in $\mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x + y, y + z, 2z)$. Verify whether W is an invariant subspace of T.
11. Let $W_1 = \text{span}\{(1,2,1)\}$ and $W_2 = \text{span}\{(2,1,1)\}$. Verify whether $W_1 + W_2$ is a direct sum.
12. Verify whether $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + y, 0)$ is a projection.
13. Verify whether $(1,2), (-2,1)$ are orthogonal in $\mathbb{R}^2$.
14. If E is an orthogonal projection of a space V on a subspace W, prove that Null $(1 - E) = W$.

(14 × 1 = 14 weightage)

Turn over

Part B (Paragraph Type)

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Let S ; $\{\alpha, \beta\}$ be a subset of V, where V is a vector space. Prove that the set of all linear combinatio of S is a subspace of V.
16. Verify whether $S = \{(x, y, xy) : x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.
17. If W_1, W_2 are subspaces of a vector space V, prove that $W_1 + W_2$ is a subspace of V.
18. Let V be a vector space of dimension n. Prove that any set of $n-1$ vectors is not a basis of V.
19. Find the matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + y, x, y +$ with respect to the ordered basis $B = \{(1,2,1), (1,1,2), (2,1,1)\}$.
20. Let $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be a basis of a vector space V and $\{f_1, f_2, \ldots, f_n\}$ be the dual basis of V^A. Pro that $\alpha = \sum_i f_i(\alpha)\,\alpha_i$ for each $\alpha \in V$.
21. Let T be a linear operator on a vector space V and for some $\alpha \in V$ let $T(\alpha) = c\alpha$. Prove that any polynomial $f, f(T)(c).\alpha$.
22. Express $\mathbb{R}^3$ as a direct sum $W_1 \oplus W_2$, where $W_1 = \text{span}\ \{(1,1,1)\}$.
23. Let T be a linear operator on a vector space $V = W_1 \oplus W_2 \oplus \ldots \oplus W_k$; where each W_i is an invari subspace for T. Prove that if each W_i is an eigen space of V then T is diagonalizable.
24. Verify whether $(x \mid y)$ given by $(x \mid y) = x_1 y_1 - x_2 y_2$ for $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$ is an in product.

(7 × 2 = 14 weighta

Part C (Essay Type)

Answer any **two** *questions.*
Each question carries weightage 4.

25. Define dimension of a vector space. Show that if W_1, W_2 are subspaces of a finite dimensi vector space, then $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.
26. Let A be an $m \times n$ matrix over a field F. Show that for any $n \times 1$ matrix X, T (X) = AX is a li transformation from the space of $n \times 1$ matrices to the space of $m \times 1$ matrices. Prove also that r T = column rank of A.
27. Define the annihilator W^0 of a subspace W. Let W_1, W_2 be subspaces of a finite dimensional s V. Prove that if $W_1^0 + W_2^0$ then $W_1 = W_2$.
28. Define projection. Show that if E is a projection on a vector space V then $V = R \oplus N$ where R i range of E and N is the null space of E.

(2 × 4 = 8 weigh