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D 33331 (Pages : 3) Name..................................

Reg. No..................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, FEBRUARY 2013

(CUCSS)

Mathematics

MT 1C 01—ALGEBRA—I

(2010 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Define isometry of R^2 and give an examples of it.
2. Find all proper non-trivial subgroups of $Z_2 \times Z_2 \times Z_2$.
3. Show that for two binary words of the same length, we have $d(u, v) = wt(u - v)$.
4. Let n be a positive integer and R be the group of all real numbers under addition. If $n\text{R} = \{nr : r \varepsilon \text{R}\}$ in a subgroup of R, compute the factor group $\frac{\text{R}}{n\text{R}}$.
5. Determine the center of A_5.
6. Define a p-group and give one example of it.
7. Obtain the class equation of a finite group G.
8. Show that every group of prime power order is solvable.
9. How many different homomorphism are there of a free group of rank 2 into S_3 ?
10. Determine the Kernel of the evaluation homomorphism $\phi_i : Q[x] \to C$.
11. If F is a field and $a \neq 0$ in a zero of $f(x) = a_0 + a_1 x + \ldots + a_n{}^{x^n}$ F [x], show that $\frac{1}{a}$ in a zero of $a_n + a_{n-1} x + .. + a_0 x^n$..
12. Let $G = \{e, a, b\}$ be a cyclic group of order 3 with identity element e. Write the element $(2e + 3a + 0b)(4e + 2a + 3b)$ in the group algebra Z_5 (G) in the form $re + sa + tb$ for $r, s, t, \varepsilon Z_5$.

Turn over

13. State division algorithm for F [x], where F is a field.

14. Give an example to show that a factor ring of an integral domain may have divisors of zero.

(14 × 1 = 14 weightag

Part B

Answer any **seven** *questions.*
Each question carries 2 weightage.

15. If m divides the order of a finite abelian group G, then show that G has a subgroup of order m.

16. Show that if a finite group G has exactly one subgroup H of a given order then H is a norm subgroup of G.

17. Show that if G has a composition series, and if N is a proper normal subgroup of G, then the exists a composition series containing N.

18. Let X be a G-set and let $x \in X$. Show that $|Gx| = (G : G_x)$.

19. Find the number of orbits in {1, 2, 3, 4, 5, 6, 7, 8} under the cyclic subgroups $\langle(1, 3, 6)\rangle$ of S_8.

20. Show that every group of order $(35)^3$ has a normal subgroup of order 125.

21. Prove that if D is an integral domain, then D [x] is an integral domain.

22. Show that the multiplicative group of all non-zero elements of a finite field is cyclic.

23. If G ={e}, the group of one element, show that R (G) is isomorphic to R for any ring R.

24. Show that a factor ring of a field is either the trivial ring of one element or is isomorphic to t field.

(7 × 2 = 14 weighta

Part C

Answer any **two** *questions.*
Each question carries 4 weightage.

25. Show that the set of all commutators of a group G generates the smallest normal group C such t

G/C is abelian. Determine the commutator subgroup C of D_4 and the factor group $\frac{D4}{C}$.

26. State the first isomorphism theorem.

Let $\phi : Z_{18} \to Z_{12}$ be the homomorphism where $\phi(1) = 10$.

(a) Find the Kernel K of ϕ.

(b) List the cosets in $\frac{Z_{18}}{K}$, showing the elements in each coset.

(c) Find the group $\phi(Z_{18})$.

(d) Give the correspondence between $\frac{Z_{18}}{K}$ and $\phi(Z_{18})$ given by the map ψ in the first isomorphism theorem.

27. State and prove first Sylow theorem. Show that a normal p-subgroup of finite group is contained in every Sylow p- subgroup of G.

28. Determine all group of order 10 up to isomorphism.

(2 × 4 = 8 weightage)