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D 33335 (Pages : 3) Name....................................

Reg. No....................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, FEBRUARY 2013

(CUCSS)

Mathematics

MT 1C 05—DISCRETE MATHEMATICS

(2010 admissions)

Time : Three Hours Maximum : 36 Weightage

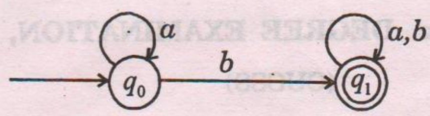
Part A (Short Answer Questions) (1–14)

Answer **all** *questions.*
Each question carries 1 weightage.

1. Let X be a set and let P (X) be its powerset. Is the relation inclusion a total order on P (X) ? Justify your answer.
2. Is union of chains a chain ? Justify your answer.
3. Define a Boolean algebra and give an example of it.
4. Is $x_1x_2'x_3x_4' + x_2x_3'x_4x_1'$ symmetric ? Justify your answer.
5. Define self complementary graphs. Give an example of it.
6. Prove that every k-regular graph with n vertices $\frac{nk}{2}$ edges.
7. Prove that every edge of a tree is a cut edge.
8. Prove that $k\left(K_{m,n}\right) = \min\{m, n\}$.
9. Prove that K_5 can't be drawn without crossings.
10. If G is a connected planar graph with at least three vertices, prove that $e(G) \leq 3n(G) - 6$, where e (G) and n (G) are the number of edges and vertices in G.
11. Let Σ be a finite alphabet and let $w \in \Sigma^+$. Prove that $\left(w^R\right)^R = w$ for all $w \in \Sigma^+$, where w^R denotes the reverse of w.
12. Find a grammar that generates the language $\{a^n b^m ; n \geq 0, m > n\}$.
13. Show that the language $\{uvu : u, v \in \{a, b\}^*, |u| = 2\}$ is regular.

Turn over

14. Find the set of strings accepted by the following deterministic finite acceptor.



(14 × 1 = 14 weight

Part B

Answer any **seven** *from the following* **ten** *questions* (15–24).
Each question has 2 weightage.

15. Let $(X, +, ., ')$ be a finite Boolean algebra.. Prove that every distinct atoms of X are mutu disjoint.

16. Prove that the set of all symmetric Boolean functions of n Boolean variables $x_1, x_2, \ldots, x_n$ subalgebra of the Boolean algebra of all Boolean functions of these variables.

17. Express the function $(xy + x'y + x'y')'(x + y)$ in their conjunctive normal form.

18. Prove that graph isomorphism relation is an equivalence relation on the set of simple graphs.

19. Prove that every closed walk contains an odd cycle.

20. Prove that the Petersen graph has girth 5.

21. Prove that a connected graph G with n vertices and $n - 1$ edges is a tree.

22. If G is a 3-regular graph, then prove that $k(G) = k'(G)$.

23. Give a grammar for the set integer numbers in Pascal.

24. Construct the deterministic acceptor that accepts the language Σ^*, where $\Sigma = \{a, b, c\}$.

(7 × 2 = 14 weighta

Part C

Answer any **two** *from the following four questions* (25–28).
Each question carries 4 weightage.

25. (a) Let $(X, \leq)$ be a poset and A be a non-empty finite subset of X. Prove that A has at least maximal element.

(b) Let $(X, +, ., ')$ be a Boolean algebra. Prove that $x + x . y = x$ for all $x, y \in X$.

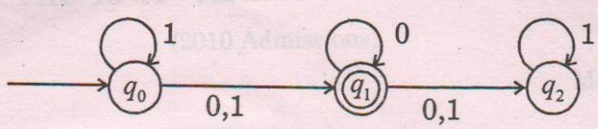
26. (a) Prove that an edge is a cut edge if and only if it belongs to no cycle.

(b) Prove that every connected graph contains a spanning tree.

27. (a) Let G be a graph with at least two vertices. Prove that every minimal disconnecting set of edges is an edge cut.

(b) If a connected plane graph has exactly n vertices, e edges and f faces, then prove that $n - e + f = 2$.

28. Construct a deterministic finite acceptor equivalent to the following non-deterministic finite acceptor.



Also find the language accepted by the above acceptor.

(2 × 4 = 8 weightage)