15P101

Name: …………………………..

Reg. No…………………………..

FIRST SEMESTER M. Sc. DEGREE EXTERNAL EXAMINATION, FEBRUARY 2016

(2015 Admission)

CC15P MT1 C01 : ALGEBRA- I

(Mathematics)

Time: Three hours | Maximum Weightage: 36

Part A

*Answer **all** the questions.*

Each question carries 1 weightage

1. Define an isometry of the Euclidean plane R^2. Give an example of an isometry.
2. Find the subgroup of Z_{18} generated by the subset {8, 6,10}
3. Show that for two binary words of the same length, d(u,v) = wt(u-v)
4. Find all proper nontrivial subgroups of $Z_2 \times Z_2 \times Z_2$
5. Describe the center of every simple abelian group.
6. Define solvable group and give one example of it.
7. Find the number of orbits in {1,2,3,4,5,6,7,8} under the cyclic subgroup $< (1\ 3\ 5\ 6) >$ of S_8.
8. Show that center of a group of order 8 is non-trivial.
9. Find all sylow 3- subgroups of S_4 .
10. Find the reduced form and the inverse of the reduced form of $a^2\, a^{-3}\, b^3\, a^4\, c^4\, c^2\, a^{-1}$
11. Define the evaluation homomorphism.
12. Write the class equation for S_3.
13. State division algorithm for F[x] where F is a field.
14. Give an example to show that a factor ring of an integral domain may have divisors of zero.

(14 x 1=14 weightage)

Part B

*Answer **any seven** questions*

Each question carries 2 weightage

15. Show that a subgroup M of a group G is a maximal normal subgroup of G iff G/M is simple.
16. Show that the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime
17. Consider the (6,3) linear code C with the standard generator matrix

$$\begin{bmatrix} 0 & & 1 \end{bmatrix}.$$

List the code words in C. How many errors can always be corrected using this code?

18. Show that the multiplicative group of all non-zero elements of a finite field is cyclic.
19. Show that there are no simple groups of order $p^r m$ where p is a prime and m < p
20. Demonstrate that $x^4 - 22x^2 + 1$ is irreducible over Q.
21. Show that for a prime p every group G of order p^2 is abelian.
22. Write all polynomials of degree ≤ 2 in $Z_2[x]$.

23. Show that a factor ring of a field is either the trivial ring of one element or is isomorphic to the field.
24. Let R be a commutataive ring with unity of prime characteristic p. Show that the map $\varphi_p : R \rightarrow R$ given by $\varphi_p(a) = G^P$ is a homomorphism.

(7 x 2=14 weightage)

Part C

*Answer **any two**, each carries 4 weightage*

25. Determine all group of order 10 up to isomorphism
26. Show that set of commutators of a group G generates the smallest normal group C such that G/C is abelian. Determine the commutator subgroup C of D_4 and the factor group D_4/C
27. Let P_1 and P_2 be sylow p- subgroups of a finite group G.Show that P_1 and P_2 are conjugate. Verify this theorem for S_4 with p = 3.
28. State and prove Cauchy's theorem. Using this prove that a finite group G is a p- group iff |G| is a power of p.

(2 x 4=8 weightage)

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