15P157

Name…………………….

Reg. No…………………..

FIRST SEMESTER M.Sc. DEGREE EXTERNAL EXAMINATION FEB. 2016

(2015 Admissions)

CC15P ST1 C05– Distribution Theory

(STATISTICS)

Time: 3 hrs. **Maximum Weightage: 36**

Part A

(Answer *all* questions. Weightage 1 for each question)

1. If X and Y are independent random variables with $P(\lambda_1)$ and $P(\lambda_1)$ respectively. Find the conditional distribution of X given $X+Y$.
2. Explain how the variates χ^2, F and t are inter-related.
3. Let $X_1, X_{2,\dots,}X_n$ be independent and identically distributed random variables with the uniform distribution on $[0, 1]$. Find the distribution of k^{th} order statistic and identify the distribution.
4. If $X \sim N(\mu, \sigma^2)$, find the distribution $Y = e^X$ and find its mean.
5. Define Pareto distribution and mention its important characteristics.
6. Define the probability generating function associated with a random variable. When will this reduce to i) characteristic function ii) moment generating function
7. Describe log normal distribution. Obtain its moment generating function and determine it's coefficient of variation.
8. What do you understand by location family. Identify a distribution belongs to location family.
9. Derive the joint distribution of $X_{(r)}$, $X_{(s)}$, the r^{th}, s^{th} order statistics.
10. Let $f(x,y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$ be the joint probability density function of X and Y. Find $E(Y/X)$.
11. Define Non central t distribution. When will this reduces to central t distribution.
12. Define χ^2- distribution. State its important uses.

(12 x 1=12 weightage)

Part B

(Answer any *eight* questions. Weightage 2 for each question)

13. $X_{(1)}$, $X_{(2)}$, $X_{(3)}$, be the order statistics of i.i.d r.v's X_1 , X_2, X_3 with common pdf

$$f(x) = \begin{cases} \beta e^{\frac{-x}{\beta}}, & x > 0 \quad \beta > 0 \\ 0, & \text{otherwise} \end{cases}$$

Show that $X_{(r)}$, $X_{(s)} - X_{(r)}$ are independent for $s > r$.

14. Let (X, Y) be a bivariate normal random variables with parameters $\mu_1, \mu_2, \sigma_1{}^2, \sigma_2{}^2$ and ρ. Let $U = aX + b, a \neq 0,$ and $V = cY + d,\ c \neq 0$. Find the joint distribution of (U, V).

15. Let $X \sim N(0,1)$, and $Y \sim \chi^2(n)$ and X and Y are independent. Obtain the distribution of $\frac{X}{\sqrt{\frac{Y}{n}}}$

16. Let $X_1, X_2, \ldots, X_n$ are i.i.d with common density

$$f(x) = \begin{cases} \frac{1}{\theta}, & \text{if } 0 \leq x \leq \theta \\ 0, & \text{otherwise} \end{cases},$$ Obtain the distribution of sample range.

17. (a). Show that for the binomial distribution $k_{r+1} = pq\frac{dk_r}{dp}, r > 1$ where k_r is the r^{th} cumulant.

    (b). If X and Y are Poisson variates, Show that conditional distribution of X /(X+Y) is binomial.

18. (a). Obtain Poisson distribution as a limiting case of Negative binomial distribution.

    (b). Define Hyper geometric distribution. Find its mean and variance.

19. Let X and Y be identically and independently distributed exponential random variable with parameter θ. Find the distribution of U where $U = \text{Min}(X_1, X_2)$.

20. For the Pareto distribution specified by $f(x) = \begin{cases} \frac{\beta\alpha^\beta}{x^{\beta+1}} & \text{if } x \geq \alpha \\ 0 & \text{if } x < \alpha \end{cases}$, Show that moment of order n exists only if $n < \beta$. Justify the use of Pareto model as a suitable model in the study of skewed data such as, the distribution of income.

21. If X has geometric distribution. Then for any two positive integers m and n

$$P\{X > m + n | X > m\} = P\{X > n\}$$

22. If X be a non-negative random variable with distribution function F(.), then show that

$$E(X) = \int_0^\infty (1 - F(x))\, dx$$

23. If X and Y are independent random variables with standard exponential distribution. Show that $Z = \frac{X}{Y}$ has an F- distribution.

24. If X_1 and X_2 are i.i.d standard exponential random variables. Find the distribution of $Y = X_1 - X_2$

(8 x 2=16 weightage)

Part C

(Answer any *two* questions. Weightage 4 for each question)

25. Define power series family. Identify three members of the family. Obtain the moment generating function of the distribution and deduce the mean and variance. Also obtain a recurrence relation satisfied by the cumulants.

26. Define non central F distribution. Let X and Y be independently distributed random variable such that X follows non central Chi-square distribution with n_1 degrees of freedom and Y follows central Chi-square distribution with n_2 degrees of freedom. Show that $F = \frac{X/n_1}{Y/n_2}$ follows non central F distribution.

27. a) Distinguish between multiple correlation and partial correlation.

    b) If (X, Y) has a bivariate normal distribution, Find $E(X|Y)$ and $E(Y|X)$

28. Let $(X_1, X_2, \ldots, X_n)$ be a random sample from $N(\mu, \sigma^2)$, $\overline{X}$ and σ^2 respectively be the sample mean and sample variance. Let $X_{n+1} \sim N(\mu, \sigma^2)$ and assume that $(X_1, X_2, \ldots, X_n, X_{n+1})$ are independent. Find the sampling distribution of $\sqrt{\frac{n}{(n+1)}}((X_{n+1} - \overline{X})|S)$

(2 x 4=8 weightage)

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