15P153

Name……………………

Reg. No…………………..

FIRST SEMESTER M.Sc. DEGREE EXTERNAL EXAMINATION FEB. 2016

(2015 Admissions)

CC15P ST1 C01 - Measure Theory and Integration

(STATISTICS)

Time: 3 Hrs. **Max. Weightage: 36**

Part A

(Answer *all* questions. Weightage 1 for each question)

1. State mean value theorem.
2. Define Riemann-Stieltjes integral
3. Define σ Field with suitable example
4. State Fatou's Lemma
5. Define integral of a measurable function.
6. Show that every subset of a set with measure '0' is measurable.
7. Define integral of a measurable function.
8. Define normed linear space
9. State Minkowski's inequality.
10. State Cartheodory extension theorem.
11. Define signed measure
12. State Monotone class lemma.

(12 x 1=12 weightage)

Part B

(Answer any *eight* questions. Weightage 2 for each question)

13. State and prove the First Mean Value Theorem.
14. Define uniform convergence. Test for uniform convergence of the sequence $\{ f_n \}$ where $f_n(x) = \frac{nx}{1+n^2x^2}$ for all real *x*.
15. State and prove Weistrass theorem
16. State and prove Monotone Convergence Theorem.
17. If $\{ f_n \}$ is a set of measurable function and g = $\lim_{n\to\infty} f_n$ then show that g is measurable function.
18. Define the integral of a measurable function. Prove or disprove | f | is a measurable implies f is measurable.
19. State and prove Hahn Decomposition Theorem.
20. Let $\{f_n\}$ be a sequence in *Lp* which converges almost everywhere to a measurable function f then prove that {fn} converges in *Lp* to f by stating necessary conditions.
21. Prove that $\int(\lim inf\, f_n d\mu) \le \lim inf \int f_n d\mu$, where $\{f_n\}$ is a non negative measurable sequence define on $(X, \boldsymbol{A})$
22. Show that a function f is measurable if and only if its positive and negative parts are measurable functions
23. State and prove Fubini's Theorem
24. Show that the intersection of any number of σ-fields is a σ-field but union of σ-fields need not be a σ-field

(8 x 2=16 weightage)

Part C

(Answer any *two* questions. Weightage 4 for each question)

25. (a) If $\{ f_n \}$ and $\{ g_n \}$ converges uniformly on a set E, Show that $\{a\, f_n + b\, g_n\}$ converges uniformly on E, where a and b are real constants

    (b) Show that $\sum_{n=1}^{\infty}(-1)^n \frac{(x^2 + n)}{n^2}$ convergence uniformly in every bounded Interval.

26. (a) State and prove Lebesgue Dominated Convergence Theorem.

    (b) If f and g are Integrable functions and α and β are real constants, show that

    $$\int (\alpha f + \beta g)\, d\mu = \alpha \int f d\mu + \beta \int g d\mu$$

27. (a) State and prove Holder's inequality and deduce Schwarz inequality from it.

    (b) Show that convergence in Lp implies convergence in measure.

28 State and prove Jordan decomposition theorem. If μ is a signed measure in a measurable space $(X, \mathcal{B})$ then show that there is a positive set A and a negative set B such that $A \cap B = \varphi$ and $X = A \cup B$

(2 x 4=8 weightage)

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