16P104 (Pages:2) Name: ……………………….

Reg. No……………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C04 – ODE AND CALCULUS OF VARIATIONS

(Mathematics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer **All** Questions

Each Question carries 1 weightage

1. Determine the nature of the point $x = 0$ for the equation $xy'' + (\sin x)\ y = 0$.
2. Find the indicial equation and its roots for $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$.
3. Define Hypergeometric series $F(a, b, c, x)$ and show that $(1 + x)^p = F(-p, b, b, -x)$.
4. What is Rodrigues formula and using the formula find the value of $p_2(x)$.
5. Define gamma function and show that $\Gamma(p + 1) = p!$.
6. Show that $\frac{d}{dx}\left(x\,J_1(x)\right) = x\,J_0(x)$
7. Describe the phase portrait of $\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = 0 \end{cases}$
8. Determine whether the function $x^2 - xy - y^2$ is positive definite, negative definite or neither.
9. State Sturm separation theorem.
10. Show that $f(x, y) = xy$ satisfy Lipschitz condition on any rectangle $a \le x \le b\ and\ c \le y \le d$
11. Find the normal form of Bessel equation $x^2y'' + xy' + (x^2 - p^2)y = 0$.
12. State Picard's theorem.
13. When the integrand in $I = \int_{x_1}^{x_2} f(x, y, y')dx$ is of the form $a(x)(y')^2 + 2b(x)yy' + c(x)y^2$, show that the Euler's equation is a second order linear differential equation.
14. Find the general solution of $\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = y \end{cases}$

(14 x 1 = 14 Weightage)

Part B

Answer **any 7** questions

Each question carries 2 weightage

15. Express $\sin^{-1} x$ in the form of a power series by solving $y' = (1 - x^2)^{-\frac{1}{2}}$ in two ways.
16. Find the general solution of $y'' + xy' + y = 0$ in the form $y = a_0y_1(x) + a_1y_2(x)$ where $y_1(x), y_2(x)$ are power series.
17. Explain least square approximation.

18. State Bessel expansion theorem and find the Bessel series for $f(x) = 1$.

19. Find the general solution of $\begin{cases} \frac{dx}{dt} = x - 2y \\ \frac{dy}{dt} = 4x + 5y \end{cases}$

20. Define Liapunov function $E(x, y)$ for the system $\begin{cases} \frac{dx}{dt} = F(x, y) \\ \frac{dy}{dt} = G(x, y) \end{cases}$. If the function has the additional property that $\frac{\partial E}{\partial x} F + \frac{\partial E}{\partial y} G$ is negative definite, then prove that the critical point (0, 0) is asymptotically stable.

21. Determine the nature and stability property of the critical point (0, 0) for $\begin{cases} \frac{dx}{dt} = -3x + 4y \\ \frac{dy}{dt} = -2x + 3y \end{cases}$

22. Let $u(x)$ be any non-trivial solution of $u'' + q(x)u = 0$ where $q(x) > 0$ for all $x > 0$. If $\int_1^\infty q(x)dx = \infty$, then prove that $u(x)$ has an infinite number of zeros on the positive x- axis.

23. Find the exact solution of the initial value problem $y' = y^2, y(0) = 1$. Starting with $y_0(x) = 1$ apply Picard's method to calculate $y_1(x), y_2(x)$ and $y_3(x)$.

24. Find the geodesis on the sphere $x^2 + y^2 + z^2 = a^2$.

(7 x 2 = 14 Weightage)

Part C

Answer **any 2** questions

Each question carries 4 weightage

25. State and prove orthogonality property of Bessel's function.

26. Find the Frobenius series solution for the equation $x^2y'' - 3xy' + (4x + 4)y = 0$.

27. Find the general solution of $\begin{cases} \frac{dx}{dt} = 3x - 4y \\ \frac{dy}{dt} = x - y \end{cases}$

28. Derive Euler's equation for an extremal and explain the different cases.

(2 x 4 = 8 Weightage)

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