16P155 (Pages:2) Name: ……………………….

Reg. No……………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P ST1 C03 – ANALYTICAL TOOLS FOR STATISTICS - II

(Statistics)

(2015 Admission Onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

(Answer *all* questions. Weightage 1 for each question)

1. Define Basis and Dimension of a vector space.
2. Define Symmetric and skew Symmetric matrices. Give an example for each.
3. Define Idempotent matrices.
4. What is minimal polynomial?
5. What is the rank factorization of a matrix?
6. If three is the Characteristic root of a matrix A, what is the corresponding root of A^4 ?
7. Define Characteristic polynomial of matrices.
8. What is Diagonalizable matrices?
9. When do you say a quadratic form $X'AX$ to be Positive definite.?
10. Define rank of a real quadratic form.
11. Define Signature of a non-singular Hermitian matrix.
12. What is g-inverse?

(12x1=12 weightage)

Part B

(Answer any *eight* questions. Weightage 2 for each question)

13. Let V be a finite dimensional vector space. Show that all bases of V have same number of elements.
14. Show that the Characteristic roots of a real symmetric matrix are real.
15. Do the vectors a_1=(3,1,2), a_2=(7,1,9), a_3=(4,1,2) form a basis for R^3?
16. If A is an m x m Idempotent matrix, then show that (a) I_m - A is also idempotent. (b) Each eigen value of A is 0 or 1.
17. Explain the Spectral decomposition of a matrix.
18. Let V be a vector space with dimension n. Show that any linear independent set in V can be extended to a basis of V.
19. If λ_1 ,λ_2,….λ_n are the characteristic roots of a matrix A, show that trace $(A^2) = \sum_{i=1}^{n} \lambda_i^2$.
20. If A is an m×m symmetric matrix, then show that the Moore-Penrose inverse A^+ is orthogonal.
21. Show that similar matrices have same minimal polynomial.
22. Show that $\overline{A}$ is the g-inverse of A if and only if A $\overline{A}$ A = A.

23. Classify the following quadratic form as Positive definite, Positive semi-definite and Indefinite $12x^2 + 2y^2 + 6z^2$ - 4yz- 4zx+ 2xy.

24. Reduce the following matrix to its Normal form and hence find its rank $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$.

(8x2=16 weightage)

Part C

(Answer any two questions. Weightage 4 for each question)

25. Let V be vector space over R with dimension n. Show that V is isomorphic to R^n.

26. If A and B are idempotent matrices, then Show that the rank of an idempotent matrix is equal to its trace.

27. (a) Determine the algebraic and geometric multiplicities of A=$\begin{bmatrix} 8 & -4 & 6 \\ 10 & -6 & 6 \\ 8 & -8 & 10 \end{bmatrix}$.

    (b) Prove that the geometric multiplicity of a characteristic root cannot exceed algebraic multiplicity of the same.

28. (a) State and prove Rank-Nullity theorem.

    (b) Find the generalized inverse of A = $\begin{bmatrix} 1 & 2 & 5 & 2 \\ 3 & 7 & 12 & 4 \\ 0 & 1 & -3 & -2 \end{bmatrix}$.

(2x4=8 weightage)

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