15P103 Name: …………………………..

Reg. No………………………….

FIRST SEMESTER M.Sc. DEGREE EXTERNAL EXAMINATION FEB. 2016

(2015 Admission)

CC15P MT1 C03: Real Analysis – I

(Mathematics)

Time :Three hours **Maximum :36 weightage**

Part A (short Answer Questions)

Answer **all** questions.

Each question has 1 weightage.

1. Define convex set. Give an example.
2. Is there a non empty perfect set in $\mathbb{R}^1$ which contains no rational number? Justify.
3. If $\gamma(t) = e^{it}$ where $0 \leq t \leq 2\pi$. Show that γ is rectifiable.
4. Define uniform continuity of a function with an example.
5. Is the subset of a connected set always connected? Justify.
6. Is set of rational numbers connected? Justify your answer.
7. Define refinement of a partition. Show that $\underline{\int_a^b} f\, d\alpha \leq \overline{\int_a^b} f\, d\alpha$
8. With an example define uniform convergence of sequence of functions.
9. State intermediate value theorem. Is the converse true?
10. Let f be monotonic on(a, b). If possible give an example of a function which is discontinuous at every irrational points in (a, b) Justify.
11. Let E be a subset of a metric space X. Show that $\bar{E}$ closed.
12. Discuss the differentiability of the modulus function in R.
13. Prove or disprove: Every continuous function is an open mapping.
14. What you mean by discontinuity of second kind. Give an example.

(14 x 1 = 14 Weightage)

Part B

Answer any **seven** from the following ten questions (15-24).

Each question has weightage 2.

15. Prove that set of rational numbers is countable.
16. Prove that there exists a real continuous function on the real line which is nowhere differentiable.
17. Show that compact subsets of a metric space is closed
18. Show that if f is continuous mapping of a metric space X into a metric space Y and if E is connected subset of X, then $f(E)$ is connected.

19. State and prove fundamental theorem of calculus.
20. Define Cantor set and prove that it is compact.
21. Show that C(X) is a complete metric space.
22. State and prove Cauchy criterion for uniform convergence.
23. Prove that a set E is open if and only if its compliment is closed.
24. State and prove L'Hospital's rule.

(7 x 2 = 14 Weightage)

Part C

Answer any ***two*** from the following four questions (25-28).
Each question has weightage 4

25. Prove that every bounded infinite subset of R^k has a limit point in R^k
26. a. State and prove Taylor's theorem.

    b. State and prove mean value theorem. Is the theorem holds for complex valued function?

27. If γ' is continuous on $[a, b]$ then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$
28. a. Suppose f is continuous 1-1 mapping of a compact metric space X onto a metric space Y. Then prove that the inverse mapping is a continuous mapping of Y onto X.

    b. Prove that compactness is the essential factor in the above result.

(2x 4 = 8 Weightage)

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