18P101 (Pages: 2) Name: ……………………

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C01 / CC17P MT1 C01 – ALGEBRA - I

(Mathematics)

(2015 Admission onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Find all abelian groups of order 16.
2. Do the rotations together with the identity map, form a subgroup of the group of plane isometries? Why?
3. Prove that a factor group of cyclic group is cyclic.
4. Compute the factor group $\frac{\mathbf{Z_2 \times Z_4}}{<(\mathbf{1,2})>}$
5. Find the number of orbits in {1,2,3,4,5,6,7,8} under the cyclic subgroup of S_8 generated by (1, 3) and (2, 4, 7)
6. State Burnside's formula.
7. If H and N are subgroups of a group G, and N is normal in G, prove that $H \cap N$ is normal in H.
8. Find the center of $S_3 \times Z_4$
9. Find the order of Sylow 3 – subgroup of a group of order 54.
10. Find the reduced form and the inverse of the reduced form of the word $a^2a^{-3}b^3a^4c^4c^2a^{-1}$
11. Give a presentation of Z_4 involving two generators.
12. Find a polynomial of degree > 0 in Z_4 [x] that is a unit.
13. Write one non zero element in End($< Z \times Z, + >$).
14. Let $\emptyset: R \to R'$ be a ring homomorphism and let N be an ideal of R. Show that $\emptyset(N)$ is an ideal of $\emptyset(R)$.

(14 × 1 = 14 Weightage)

Part **B**

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Show that if m divides the order of a finite abelian group G, then G has a subgroup of order m.

16. Show that if G is nonabelian, then the factor group ${}^{G}/_{Z(G)}$ is not cyclic.

17. State and prove Third isomorphism theorem.

18. Find isomorphic refinements of the two series $\{0\} < \langle 18\rangle < \langle 3\rangle < \mathbb{Z}_{72}$ and $\{0\} < \langle 24\rangle < \langle 12\rangle < \mathbb{Z}_{72}$

19. State and prove second sylow theorem.

20. Show that every group G' is a homomorphic image of a free group G

21. Show that the presentation $(a, b : a^3 = 1, b^2 = 1, ba = a^2 b)$ gives a nonabelian group of order 6

22. Let D be an integral domain and x an indeterminate. Describe the units in $D[x]$

23. Give the addition and multiplication tables for the group algebra $Z_2 G$, where $G = \{e, a\}$ is a cyclic group of order 2

24. Let R be a commutative ring and let a ∈ R.Show that $I_a = \{x \in R : ax = 0\}$ is an ideal of R

(7 × 2 = 14 Weightage)

Part **C**

Answer any ***two*** questions. Each question carries 4 weightage.

25. (a) Let H be a subgroup of G. Then show that the left coset multiplication is well defined by the equation (aH). (bH) = (ab)H if and only if H is a normal subgroup of G

    (b) Prove that $\frac{Z}{nZ} \cong Z_n$

26. (a) Let G be a group and H be a subgroup of G. Prove that the set of all left cosets of H in G form a G set

    (b) Let X be a G set and let x ∈ X. Prove that $|Gx| = (G: G_x)$

27. (a) Prove that for a prime p, every group G of order p^2 is abelian.

    (b) Find the decomposition of D_4 into conjugate classes.

28. (a) State and prove Eisenstein criteria of irreducibility over Q

    (b) Find all prime numbers p such that $x + 2$ is a factor of $x^4 + x^3 + x^2 - x + 1$ in $Z_p[x]$

(2 × 4 = 8 Weightage)

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