18P105 (Pages: 2) Name: …………………….

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC15P MT1 C05 / CC17P MT1 C05 – DISCRETE MATHEMATICS

(Mathematics)

(2015 Admissions onwards)

Time: Three Hours Maximum: 36 Weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Characterize atoms of a power set Boolean algebra.
2. Obtain the conjunctive normal form of the Boolean expression $xy'(x' + y' + xy)$
3. Define Boolean algebra. Prove that $x + x.y = x \quad \forall\, x, y \in X$ where $(X, +, ., ')$ is a Boolean algebra.
4. Define a strict partial order. If P is a partial order on the set X, show that $\mathrm{P} - \{(\mathrm{x}, \mathrm{x}) \colon \mathrm{x} \in \mathrm{X}\}$ is a strict partial order.
5. For any simple graph G, show that $\surd(G) = \surd(G^c)$
6. If G is simple and $\delta \geq \frac{n-1}{2}$, then show that G is connected.
7. Prove that a connected graph G with at least two vertices contains at least two vertices that are not cut vertices.
8. If $(G) \geq 2$, then prove that G contains a cycle.
9. Show that the number of edges of a simple graph with ω components cannot exceed $\frac{(n-\omega)(n-\omega+1)}{2}$
10. Define connectivity and edge connectivity. Give an example.
11. Let $\sum = \{a, b\}$ and L = $\{a^n b^m : n \geq 0, m > n\}$. Find a grammar that generates L.
12. If $\sum = \{0,1\}$, design an NFA to accept set of strings ending with two consecutive zeroes.
13. Find an NFA which accepts the set of all strings containing ‘aabb’ as a substring.
14. Find a DFA for the language L= $\{a^n \colon n\ is\ odd. n \neq 3\}$

(14 x 1 =14 Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Let $(X, +, ., ')$ be a finite Boolean algebra.Then prove that every element of X can be uniquely expressed as the sum of atoms in it.
16. State and prove Stone representation theorem for finite Boolean algebras.
17. Prove that the relation $' \leq '$ defined by $x \leq y$ if $x.y' = 0$ makes the underlying set of Boolean algebra into a lattice.
18. Prove that a graph is bipartite iff it contains no odd cycles.
19. Prove that in a connected graph G with at least 3 vertices, any two longest paths have a vertex in common.
20. Prove that $\kappa(G) \leq \lambda(G) \leq \delta(G)$for any loopless connected graph G.
21. State and prove Euler's formula.
22. Show that an edge in a simple graph is a cut edge iff it belongs to no cycles.
23. Define NFA and DFA.
24. Design a DFA that accepts the language $L = \{ab^n a^m, n \geq 2, m \geq 3\}$

(7 x 2 =14 Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. i) Let $(X, \leq)$ be a poset and A be a non-empty finite subset of X. Then prove that A has at least one maximal element.

    ii) Prove that a finite non-empty subset of a poset has a maximum element iff it has a unique maximal element.
26. State and prove Whitney's theorem. Also show that a graph G with at least 3 vertices is 2-connected iff any two vertices of G lie on a common cycle.
27. Prove that a connected graph G with atleast two vertices is a tree iff its degree sequence $(d_1, d_2, \dots \dots . d_n)$ satisfies the condition $\sum_{i=1}^{n} d_i = 2(n-1)$
28. Show that the grammar G with $\sum = \{a, b\}$ and productions $S \rightarrow SS,\ S \rightarrow \lambda,$ $S \rightarrow aSb,\ S \rightarrow bSa,$ generates the languge $L = \{w: n_a(w) = n_b(w)\}$ where $n_a(w)$ and $n_b(w)$ are number of $a's$ and $b's$ in w respectively.

(2 x 4 = 8 Weightage)

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