18P104 (Pages: 2)

Name:......................

Reg.No:....................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2018

(Regular/Supplementary/Improvement)

(CUCSS-PG)

CC17P MT1 C04 NUMBER THEORY

(Mathematics)

(2017 Admission onwards)

Time :Three Hours Maximum : 36 weightage

Part A

Answer ***all*** questions. Each question carries 1 weightage.

1. Define Möbius function $\mu(n)$ and show that $\sum_{d^2/n} \mu(d) = \mu^2(n)$
2. Let $d(n)$ denotes the number of positive divisors of n. Prove that $d(n)$ is odd if and only if n is square.
3. If f is multiplicative, then $F(n) = \prod_{d/n} f(d)$ is multiplicative. Prove or disprove.
4. Find all integers n such that $\phi(n) = n/2$
5. Assume f is multiplicative, then prove that $f^{-1}(p^2) = (f(p))^2 - f(p^2)$ for every prime p.
6. Prove that for $x \geq 1$, $\sum_{n \leq x} \mu(n)[\frac{x}{n}] = 1$
7. Let $f(x) = x^2+x+41$. Find the smallest integer $x \geq 0$ for which $f(x)$ is composite.
8. State and prove Legendre's identity.
9. Define Legendre's symbol.
10. If p is prime, prove that $\sum_{r=1}^{\text{p-1}}(r/p) = 0$
11. Prove that 5 is a quadratic residue of an odd prime p if $p \equiv \pm 1(\mod 10)$.
12. Derive Selberg identity.
13. Find the inverse of the matrix $\begin{pmatrix} 1 & 3 \\ 4 & 3 \end{pmatrix}$ mod 5
14. What is a cryptosystem?

($14 \times 1 = 14$ Weightage)

Part B

Answer any ***seven*** questions. Each question carries 2 weightage.

15. Prove that every number of the form $2^{a-1}(2^a - 1)$ is perfect if $2^a - 1$ is prime.
16. If f is completely multiplicative, prove that $(f.g)^{-1} = f.g^{-1}$ for every arithmetical fuction g with $g(1) \neq 0$ where $f.g$ denotes the ordinary product, $(f.g)(n) = f(n)g(n)$
17. State and prove Euler's summation formula.
18. If $A(x) = \sum_{n \leq x} \frac{\mu(n)}{n}$, then prove that the relation $A(x) = o(1)$ as $x \to \infty$ implies the prime number theorem.
19. State and prove Abel's identity.
20. Determine whether 219 is a quadratic residue or nonresidue mod 383
21. State and prove Euler's criterion for Legendre's symbol.
22. State and prove reciprocity law for Jacobi symbols.
23. Explain briefly about classical cryptosystem.
24. How will you authenticate a message in public key cryptosystem.

($7 \times 2 = 14$ Weightage)

Part C

Answer any ***two*** questions. Each question carries 4 weightage.

25. If $x \geq 1$ then prove that

    (a) $\sum_{n \leq x} \frac{1}{n} = \log x + C + O(\frac{1}{x})$

    (b) $\sum_{n > x} \frac{1}{n^s} = O(x^{1-s})$ if $s > 1$

26. Let $\{a(n)\}$ be a non-negative sequence such that: $\sum_{n \leq x} a(n)[\frac{x}{n}] = x \log x + O(x)$ for all $x \geq 1$. Prove that there is a constant $B > 0$ such that: $\sum_{n \leq x} a(n) \leq nB(x)$ for all $x \geq 1$.
27. Determine those odd primes p for which 3 is a quadratic residue mod p and those for which it is a non-residue.
28. State and prove Gauss' Lemma.

($2 \times 4 = 8$ Weightage)

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