19P158 (Pages: 2) Name: ……….……………

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCSS PG)

CC19P MST1 C03 – DISTRIBUTION THEORY

(Statistics)

(2019 Admission Regular)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Define log-normal distribution. Also determine mean and variance of the distribution.
2. Obtain the moment generating function of the Normal distribution $N(\mu, \sigma^2)$
3. Write down the probability mass function of the negative binomial distribution. Also obtain (a) mean; and (b) variance.
4. Obtain the characterization of Weibul distribution.
5. $X_1, X_2, \dots, X_n$ are independent geometric random variables, identically distributed with parameter p. Obtain the distribution of $X_{(1)} = \min(X_1, X_2, \dots, X_n)$
6. If X follow the uniform distribution, $U(0,1)$, obtain the distribution of $Y = -2\log X$
7. If $X \sim Chi\ Square\ (n)$ and $Y \sim Chi\ Square\ (m)$, obtain the distribution of X/Y and X+Y

(2 x 4 = 8 Weightage)

PART B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Write down the differential equation satisfied by the Pearson system of distributions. What is the basis for classification of member of the family into various type? Give an example.
9. Let X and Y be independent random variables following the negative binomial distributions, $NB(r_1, p)$ and $NB(r_2, p)$ respectively. Show that the conditional probability mass function of X given $X + Y = t$ is hypergeometric.
10. Define the hypergeometric distribution. Show that Hypergeometric distribution tends to the binomial distribution.
11. Derive the distribution of sample mean and sample variance of a sample drawn from Normal population.

12. Define mixture distributions. Obtain the expression for the mean and variance of the mixture distribution in terms of the mean and variance of the component distributions.
13. Show that if $E(X^2) < \infty$, then prove that $V(X) = V\big(E(X|Y)\big) + E\big(V(X|Y)\big)$
14. If X and Y are independent exponential (β) random variables. Obtain the distribution of X+Y

(4 x 3 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage.

15. a) Obtain moment generating function of Gamma distribution. Establish the additive property of Gamma distribution.

    b) If X has a standard Cauchy distribution, find the distribution of $Y = |X|$
16. Find the joint pdf of the range w and midpoint m in random samples of size n from $U(-\frac{1}{2}, \frac{1}{2})$. Hence or otherwise find the pdf of m and its variance.
17. In sampling from a normal population, show that the sample mean $\overline{X}$ and the sample variance S^2 are independently distributed.
18. Define the non-central $Chi - square$ statistic and derive its distribution. Obtain the expression for mean and variance. Also describe the applications of the distribution.

(5 x 2 = 10 Weightage)

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