19P159 (Pages: 2) Name: ……………………

Reg. No…………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCSS-PG)

CC19P MST1 C04 – PROBABILITY THEORY

(Statistics)

(2019 Admissions)

Time: Three Hours Maximum: 30 Weightage

Part A

Answer any ***four*** questions. Each question carries 2 weightage.

1. Show that the intersection of arbitrary number of fields is a field. Also show that union of two fields is not a field.
2. State and prove Jensen's inequality.
3. If $\{A_n, n \geq 1\}$ is a sequence of independent events and if $\sum_{n=1}^{\infty} P(A_n) = \infty$, then show that $P(\overline{\lim} A_n) = 1$
4. Define convergence in r^{th} mean. Show that convergence in probability does not imply convergence in r^{th} mean.
5. If $X_n \xrightarrow{L} C$ (a constant), then show that $X_n \xrightarrow{P} C$
6. State Kolmogorov Three Series theorem.
7. State Lindberg Feller form of Central Limit Theorem.

(4 x 2 = 8 Weightage)

Part B

Answer any ***four*** questions. Each question carries 3 weightage.

8. Define vector random variable. Also explain the sigma field induced by a sequence of random variables.
9. State Inversion formula. Find the probability density function corresponding to the characteristic function $\varphi(t) = e^{-\frac{t^2}{2}}$
10. State and prove Kolmogorov 0-1 law.
11. Define almost sure convergence of random variables. State and prove necessary and sufficient condition for almost sure convergence.
12. State and prove Markov's inequality. Using that show that r^{th} mean convergence imply convergence in probability.

13. Write down the sufficient conditions for a sequence of random variables to follow the weak law of large numbers. Let $\{X_n, n \geq 1\}$ be a sequence of independent random variables with $P[X_n = \pm 2^n] = 2^{-(2n+1)}, n \geq 1, P[X_n = 0] = 1 - 2^{-2n}$. Check whether the sequence obeys the weak law of large numbers or not.

14. a) Show that characteristic function is uniformly continuous.

    b) If X is a random variable with characteristic function $\varphi(t)$ then show that $\varphi(t)$ is real if and only if probability distribution of X is symmetric.

(4 x 3 = 12 Weightage)

Part C

Answer any ***two*** questions. Each question carries 5 weightage.

15. Write short notes on

    (i) Borel field.
    (ii) Lebesgue measure.
    (iii) Counting measure.
    (iv) Lebesgue Stieltjes measure.

16. State and prove inversion theorem.
17. State and prove Levy continuity theorem.
18. State and prove Khintchin's weak law of large numbers.

(2 x 5 = 10 Weightage)

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