19P101 (Pages: 2) Name: ………………….

Reg. No………………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCSS PG)

CC19P MTH1 C01 - ALGEBRA-I

(Mathematics)

(2019 Admission Regular)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Are the groups $\mathbb{Z}_2 \times \mathbb{Z}_{12}$ and $\mathbb{Z}_4 \times \mathbb{Z}_6$ isomorphic? Justify.
2. Find the order of $(\mathbb{Z}_{12} \times \mathbb{Z}_{18})/< (4,3) >$
3. $\mathbb{R}$ / $\mathbb{Z}$ under addition has no element of order 2. Justify.
4. Give isomorphic refinements of $\{0\} < 60\ \mathbb{Z} < 20\ \mathbb{Z} < \mathbb{Z}$ and $\{0\} < 245\mathbb{Z} < 49\ \mathbb{Z} < \mathbb{Z}$
5. Show that every group of order 45 has a normal subgroup of order 9
6. Find all zeros of $x^5 + 3x^3 + x^2 + 2x$ in $\mathbb{Z}_5$
7. If F is a field then F[x] is a field. Justify.
8. Is $f(x) = x^2 + 8x - 2$ irreducible over $\mathbb{Q}$?

(**8 × 1 = 8 Weightage**)

PART- B

Answer any ***two*** questions from each unit. Each question carries 2 weightage.

UNIT 1

9. Show that if m divides the order of a finite abelian group G then G has a subgroup of order m.
10. Show that M is a maximal normal subgroup of G if and only if G\M is simple. List the maximal normal subgroups of $\mathbb{Z}_8$ with respect to addition modulo 8.
11. Prove that $|Gx|$=$(G: G_x)$ where X is a G- set and $x \in X$

UNIT 2

12. State and prove third Sylow theorem.
13. Show that every group of order 255 is abelian.
14. Define solvable group. Is S_3 solvable? Justify.

UNIT 3

15. Show that the polynomial $\Phi_p(x) = \frac{x^p - 1}{x-1}$ is irreducible over $\mathbb{Q}$ for any prime p

16. Prove that if G is a finite subgroup of the multiplicative group $\langle F^*, \cdot \rangle$ of a field F then G is cyclic.

17. Give the addition and multiplication table for the group algebra $\mathbb{Z}_2 G$ where G = {e, a}

$(\mathbf{6 \times 2 = 12\ weightage})$

PART C

Answer any ***two*** questions**.** Each question carries 5 weightage.

18. (a) State and prove Burnsides' formula.

    (b) How many distinguishable necklaces (with no clasp) can be made using seven different colored beads of the same size?

19. (a) Prove that if G is a group of order p^n and X is a finite G-set, then $|X| \equiv |X_G|\ mod\ p$

    (b) State and prove Cauchy's theorem.

20. (a) State and prove Eisenstein criterion.

    (b) Verify whether $8x^3 + 6x^2 - 9x + 24$ is irreducible over $\mathbb{Q}$

21. Determine all subgroups of order 10 up to isomorphism.

$(\mathbf{2 \times 5 = 10\ Weightage})$

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