19P103 (Pages: 2) Name: ……….……………

Reg. No……….…………...

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCSS PG)

CC19P MTH1 C03 – REAL ANALYSIS-I

(Mathematics)

(2019 Admission Regular)

Time: Three Hours Maximum: 30 Weightage

PART A

Answer ***all*** questions. Each question carries 1 weightage.

1. Prove that the set of all rational numbers is countable.
2. Prove that union of an infinite collection of closed sets need not be closed.
3. If E is an infinite subset of a compact set K, then prove that E has a limit point in K
4. If f is a continuous mapping of the closed unit interval $[0\,,1]$ into itself, then prove that $f(x)=x$ for at least one $x \epsilon [0\,,1]$
5. Show that L'Hospital's rule fails to hold for vector valued functions.
6. Let f be a bounded real function defined on $[a\,,b]$ and let α be monotonically increasing function on $[a\,,b]$. Then prove that $\int_{-a}^{b} f\,d\alpha \leq \int_{a}^{-b} f\,d\alpha$
7. If $f \epsilon \mathcal{R}(\alpha)$ on $[a\,,b]$, then prove that $|f| \epsilon \mathcal{R}(\alpha)$ and $|\int_{a}^{b} f\,d\alpha| \leq \int_{a}^{b}|f|d\alpha$
8. Let $f_n(x)=\frac{\sin nx}{\sqrt{n}}$ (x real, n $= 1,2,3,\ldots$), then prove that $\{f_n{}'\}$ does not converge to f'

(8 x 1 = 8 Weightage)

PART B

Answer any ***two*** questions **f**rom each unit. Each question carries 2 weightage.

UNIT I

9. Prove that a finite point set has no limit points.
10. Prove that a set E is open if and only if its complement is closed.
11. If f is a continuous mapping of a metric space X into a metric space Y, and if E is a connected subset of X, then prove that $f(E)$ is connected.

UNIT II

12. State and prove chain rule of differentiation.
13. If f is continuous on $[a\,,b]$, then prove that $f \epsilon \mathcal{R}(\alpha)$ on $[a\,,b]$
14. State and prove the fundamental theorem of calculus.

UNIT III

15. State and prove the Cauchy criterion for uniform convergence of a sequence of functions defined on E

16. Prove that there exists a real continuous function on the real line which is nowhere differentiable.

17. Define equicontinuous family of functions. If K is a compact metric space, if $f_n \epsilon \mathcal{C}(K)$ for $n = 1,2,3, \ldots$, and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K

(6 x 2 = 12 Weightage)

PART C

Answer any ***two*** questions. Each question carries 5 weightage**.**

18. i) Prove that continuous mapping of a compact metric space X into a metric space Y is uniformly continuous.

    ii) Give an example of a continuous function on $(0\ ,1)$ which is not uniformly continuous.

19. i) If f is differentiable on [a , b], then prove that f' cannot have simple discontinuities on [a, b]

    ii) State and prove Taylor's theorem.

20. i) Prove that a continuously differentiable curve γ defined on an interval $[a, b]$ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt$

    ii) Is γ defined on $[0\ , 2\pi]$ by $\gamma(t) = e^{2it}$ rectifiable?

21. If f is a continuous complex function on $[a\ , b]$, then prove that there exists a sequence of polynomials $P_n(x)$ such that $\lim_{n\to\infty} P_n(x) = f(x)$

(2 x 5 = 10 Weightage)

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