Reg. No.....

# FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2016

(Regular/ Supplementary/ Improvement) (CUCBCSS - UG)

CC15UMAT1C01- MATHEMATICS (Mathematics Complementary course) (2015 Admission onwards)

Time: 3 hrs

Max Mark: 80

### I Answer all questions

(12x1 = 12 Marks)

- 1. The function  $f(x) = \frac{|x|}{x}$  is not continuous at  $x = \dots$
- 2. Evaluate  $\lim_{x \to \infty} \frac{5x^2 + 8x 3}{3x^2 + 2}$
- 3. At what points do the graph of the function  $f(x) = x^2 + 4x 1$  has horizontal tangents
- 4. What is the parametric form of  $x^2 + y^2 = 4$
- 5. How fast does the area of the circle change with respect to the diameter when the diameter is 8m.
- 6. The interval on which the function f given by  $f(x) = 2x^3 3x^2 36x + 7$  decreasing
- 7. The horizontal asymptote for the curve  $y = \frac{1}{x}$  is......
- 8. If f(x) = -9 on [-1,6],  $\int_{-1}^{6} f(x) dx = \dots$
- 9. State the mean value theorem for the definite integrals.
- 10. The dominant terms of  $f(x) = \frac{x^2 4}{x 1}$  are ......
- 11. Find dy/dx if  $y = \int_0^{\tan x} \frac{dt}{1+t^2}$
- 12. Evaluate  $\int_{-\pi}^{\pi} \sin^2 x \, dx$

# II Answer any nine questions

(9x2 = 18 Marks)

- 13. Find  $\lim_{x \to 0^+} \frac{|x|}{x}$  and  $\lim_{x \to 0^-} \frac{|x|}{x}$
- 14. State Sandwich theorem and use it to find  $\lim_{x \to 0} g(x)$  if  $3 x^2 \le g(x) \le 3 \sec x \ \forall x$
- 15. Find the linearization of  $f(x) = x^3 x$  at x = 1
- 16. Evaluate  $\lim_{x \to 1} \left( \frac{1-x}{\ln x} \right)$
- 17. Find the average value of  $f(x) = 3x^2 3$  on [0,1]
- 18. Find the function f(x) whose derivate is  $\sin x$  and whose graph passes through the point (0, 2)
- 19. Find the interval in which  $f(x) = x^2$  is decreasing

- 20. State why the function  $f(x) = x^{\frac{2}{3}}$ ; [-1,8] does not satisfy the conditions of Mean value theorem
- 21. Express the following limit as a definite integral  $\lim_{\|P\|\to 0} \sum_{k=1}^n \frac{1}{1-C_k} \Delta x_k$  where P is a partition of [2,3].
- 22. If  $\int_0^3 f(z)dz = 3$ ,  $\int_0^4 f(z)dz = 7$ , and f is continuous. Find  $\int_4^3 f(z)dz$ .
- 23. The curve  $y = ax^2 + bx + c$  passes through the point (1, 2) and is tangent to the line y = x at the origin. Find a, b, and c
- 24. The radius of a circle is increased from 2m to 2.02m. Estimate the resulting change in the area.

# III Answer any six questions

(6x5 = 30 Marks)

- 25. Using  $\varepsilon \delta$  definition, prove that  $\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$
- 26. Is there a real number that is one less than its fifth power?
- 27. Prove that if a function is differentiable at x = c, then it is continuous at x = c. Is the converse true? Justify?
- 28. Verify mean value theorem for the function  $f(x) = \ln x$  on the interval [1, e]
- 29. Prove that for the curve  $y = c sin \frac{x}{a}$ , every point at which it meets the x axis is a point of inflexion.
- 30. Find all the asymptotes of the curve  $y = 2 + \frac{\sin x}{r}$
- 31. Show that if f is continuous on [a, b],  $a \ne b$  and if  $\int_a^b f(x) dx = 0$ , then f(x) = 0 at least once in [a, b]
- 32. Evaluate  $\int_0^1 x \, dx$  using Riemann definition
- 33. Find the area of the region enclosed by the parabola  $x = y^2$  and the line x = y + 2

# IV Answer any two questions

(2x10 = 20 Marks)

- 34. Using the definition of limit prove that  $\lim_{x \to 1} f(x) = 2$  if  $f(x) = \begin{cases} 4 2x, & x < 1 \\ 6x 4, & x \ge 1 \end{cases}$
- 35. Graph the function  $y = x^{\frac{5}{3}} 5x^{\frac{2}{3}}$
- 36. Use definite integral to estimate the sum of the square roots of the first n positive integers  $\sqrt{1} + \sqrt{2} + \dots + \sqrt{n}$